Exponential Rules

$$u^{-m} = \frac{1}{u^m} \qquad \& \qquad u^{\frac{m}{n}} = \sqrt[n]{u^m} \qquad \& \qquad u^0 = 1$$

$$u^m . u^n = u^{m+n} \qquad \& \qquad \frac{u^m}{u^n} = u^{m-n}$$

$$(u^m)^n = u^{m.n}$$

$$u^m . v^m = (u . v)^m$$

$$(-u)^n = u^n \text{ If n is even,} \qquad (-u)^n = -u^n \text{ if n is odd}$$

Negative exponent is called Reciprocal:

Ex.: $\qquad\qquad\qquad\qquad\qquad 5^{-3} = \frac{1}{5^3} = \frac{1}{125} \qquad\qquad\qquad x^{-n} = \frac{1}{x^n}$

Rational and Irrational or Surd:

If the power of the term under the radicand is a multiple of the index then the result is Rational.

Ex.: $\qquad\qquad \sqrt[3]{27} = \sqrt[3]{3^3} = 3^{\frac{3}{3}} = 3 \qquad\qquad \sqrt{0.25} = 0.5 \qquad\qquad \sqrt[3]{x^6} = x^{\frac{6}{3}} = x^2$

Otherwise the result is "Irrational or Surd".

Ex.: $\qquad\qquad \sqrt{3} \qquad\qquad \sqrt[3]{4} \qquad\qquad \frac{2}{\sqrt{5}} \qquad\qquad \sqrt{\frac{3}{10}}$

Calculating expressions with radicands, it is recommended to not leave the denominators of fractions with radicand, it's more convenient, and comprehensive if rationalized.

Rationalization:

Ex.: $\quad$ *Rationalize:* $\qquad\qquad a. \ \frac{2}{-\sqrt{5}} \qquad\qquad b. \ \sqrt{\frac{3}{10}} \qquad\qquad c. \ \frac{2}{\sqrt[3]{7}}$

Solution: multiply both numerator and denominator to denominator:

$\quad a. \ \ \frac{2}{-\sqrt{5}} = \frac{-2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = -\frac{2\sqrt{5}}{5} \qquad b. \ \sqrt{\frac{3}{10}} = \frac{\sqrt{3}}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{30}}{10} \qquad c. \ \frac{2}{\sqrt[3]{7}} = \frac{2}{\sqrt[3]{7}} \times \frac{\sqrt[3]{7^2}}{\sqrt[3]{7^2}} = \frac{2\sqrt[3]{7^2}}{\sqrt[3]{7^3}} = \frac{2 \times 7^{\frac{2}{3}}}{7^{\frac{3}{3}}} = \frac{2\sqrt[3]{49}}{7}$

If the denominator is 2 terms, then we multiply it by its conjugate.

Ex.: $\qquad\qquad$ *Rationalize:* $\qquad\qquad a. \ \frac{3}{2+\sqrt{5}} \qquad\qquad b. \ \frac{\sqrt{2}}{\sqrt{5}-\sqrt{3.}}$

Solution:

$\quad a. \ \ \frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{3 \times (2-\sqrt{5})}{4-5} = \frac{6-3\sqrt{5}}{-1} = 3\sqrt{5} - 6$

$\quad b. \ \ \frac{\sqrt{2}}{\sqrt{5}-\sqrt{3.}} = \frac{\sqrt{2}}{\sqrt{5}-\sqrt{3.}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{2}(\sqrt{5}+\sqrt{3})}{5-3} = \frac{\sqrt{10}+\sqrt{6}}{2}$